Adiabatic quantum pumping at the Josephson frequency

S. Russo,[1] J. Tobiska,[2] T.M. Klapwijk,[1] and A.F. Morpurgo[1]

[1] *Kavli Institute of Nanoscience, Faculty of Applied Science, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands*

[2] *NTT Basic Research Laboratories, Atsugi, Japan*

(Dated: February 5, 2008)

We analyze theoretically adiabatic quantum pumping through a normal conductor that couples the normal regions of two superconductor/normal metal/superconductor Josephson junctions. By using the phases of the superconducting order parameter in the superconducting contacts as pumping parameters, we demonstrate that a non zero pumped charge can flow through the device. The device exploits the evolution of the superconducting phases due to the ac Josephson effect, and can therefore be operated at very high frequency, resulting in a pumped current as large as a few nanoAmperes. The experimental relevance of our calculations is discussed.

PACS numbers: 72.10.-d,73.23.-b,74.45.+c

In a mesoscopic conductor in which electrons move phase coherently, a direct current can flow in response to a slowly varying periodic perturbation and in the absence of any applied bias. This phenomenon, known as quantum pumping, was first noticed by Thouless[1], who analyzed theoretically the response of an electron system to a "traveling" periodic potential $U(x - vt)$. The occurrence of quantum pumping requires that the periodic perturbation consists of at least two independent oscillating parameters $X_1(t)$ and $X_2(t)$, and that the trajectory representing the perturbation in the parameter space (X_1, X_2) encloses a finite area[2, 3]. Indeed, the proposal of Thouless satisfies these requirements since even the simplest travelling periodic potential $U(x - vt) = U_0 \sin(x - vt)$ can be written as $U(x - vt) = X_1(t) \sin(\frac{2\pi x}{\lambda}) + X_2(t) \cos(\frac{2\pi x}{\lambda})$, with $X_{1,2}(t) = X_{1,2} \cos(\frac{2\pi t}{\tau} + \phi_{1,2})$. When the cyclic perturbation is slower than the electron dwell time in the conductor, adiabatic pumping occurs and the system remains in thermodynamic equilibrium. In this case, the pumped charge can be expressed as a function of the scattering matrix and of its derivatives with respect to the pumping parameters X_1 and X_2 [3].

Attempts to investigate experimentally adiabatic quantum pumping have been made using electrostatically defined quantum dots in GaAs-based heterostructures [4]. In such a system, pumping is induced by oscillating voltages applied to the gate electrodes defining the dot. Although signatures of pumping signals may have been observed, the experiments are hindered by rectification effects originating from parasitic coupling of the *ac* signal applied to the gates [5]. Many other proposals of devices have been put foward in the literature, in which different physical quantities have been used as pumping parameters such as a time-varying magnetic field, the height of a tunnel barrier, etc.[6, 7, 8]. Often, however, these proposals do not consider the difficulties involved in the experimental realization.

Here we demonstrate theoretically the occurrence of pumping in a system of electrons and holes in a metallic conductor coupled to superconductors, where the pumping parameters are the phases of the superconducting order parameters in two different superconducting contacts. This system can operate at very high frequency without the need of feeding microwave radiation, simply by exploiting the evolution of the superconducting phases due to the *ac* Josephson effect. As a consequence, measurable pumped currents as large as a few nanoAmperes, can be expected, while avoiding spurious effects that affected previous experiments.

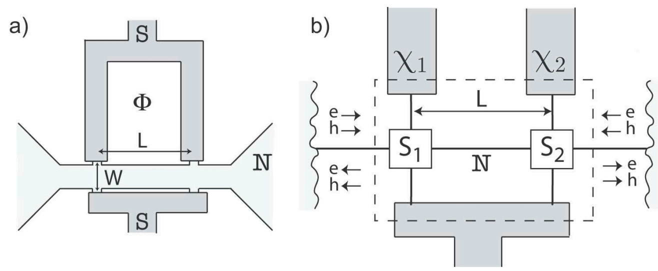

FIG. 1: a) Layout of the proposed quantum pump, consisting of two SNS Josephson junctions in a SQUID geometry with a common normal region (The dark grey areas represent the superconductor electrodes and light grey the normal conductor). b) Diagramatic representation of the scattering matrix of the device.

Fig. 1a shows a schematic representation of the circuit that we propose. Two superconducting/normal metal/superconducting (SNS) Josephson junctions are connected in parallel via a superconducting ring, and their N regions are additionally coupled by a normal metal bridge. Andreev reflection [9] of electrons and holes takes place at each NS interface, resulting in a phase shift of the particle wavefunction which at the Fermi energy is given by $\pm\chi_{1,2}$, the phase of the superconductor order parameter at the two different superconducting contacts (the sign - is for reflection from hole to electron; the sign + for the reverse process).

Hence, the total scattering matrix (S_{tot}) of the normal metal bridge connecting the left and right reservoirs depends on the quantities $X_1 = e^{i\chi_1}$ and $X_2 = e^{i\chi_2}$. We want to see if a direct current can flow in the normal metal bridge when X_1 and X_2 are used as pumping parameters.

The appealing aspect of such a device is the way in which the pumping parameters can be driven at high frequency, and their relative phase controlled. Specifically, the pumping parameters become time-dependent when a constant voltage V_{dc} is present across the SNS junctions (e.g., by biasing the junctions with a current higher than their critical current), since then $X_1(t)$ and $X_2(t) \propto e^{i\frac{2eV_{dc}}{\hbar}t}$ owing to the *ac* Josephson effect [10]. Similarly to what happens in superconducting quantum interference devices (SQUIDS), the phase difference φ between X_1 and X_2 can be easily controlled by applying a magnetic flux Φ to the superconducting loop, so that $\chi_2 = \chi_1 + \varphi$, with $\varphi = 2\pi\Phi/\Phi_0$ ($\Phi_0 = h/2e$).

To demonstrate the occurrence of pumping, we model the system in the simplest possible way. We confine ourselves to the case of a fully phase coherent system at $T = 0$ K. The normal conductor is taken to consist of one channel supporting ballistic motion, and the separation between the Josephson junctions L. The N/S interfaces are all supposed to be perfectly transparent, *i.e.* the probability of Andreev reflection is unity.

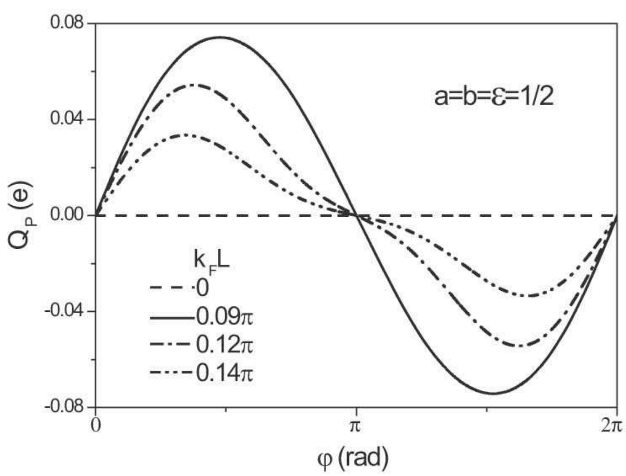

FIG. 2: Pumped charge per cycle Q_P as a function of phase difference φ between the pumping parameters, for the case $a = b = \varepsilon = 1/2$ and different values of $k_F L$.

The pumped current is equal to the charge pumped per cycle, multiplied by the pumping frequency. The calculation of the charge pumped per cycle follows the approach developed by Brouwer[3], modified to take into account the presence of the superconducting electrodes[8]. The relation between the charge $Q_{P,m}$ pumped in one of the two reservoirs (labeled by $m = 1, 2$) and the scattering matrix reads:

$$Q_{P,m} = e \int_0^\tau dt \left(\frac{dn_m}{dX_1}\frac{dX_1}{dt} + \frac{dn_m}{dX_2}\frac{dX_2}{dt} \right), \qquad (1)$$

in which:

$$\frac{dn_m}{dX_{1,2}} = \frac{1}{2\pi}\sum_{i,j} \gamma_{ij} \mathrm{Im} \frac{\partial (S_{tot})_{ij}}{\partial X_{1,2}} (S_{tot})^*_{ij}, \qquad (2)$$

where τ is the period of one pumping cycle ($\tau = \frac{2\pi}{\omega_J}$, with $\omega_J = \frac{2eV_{dc}}{\hbar}$). In Eq. 2, the sum over i extends to the electron and hole (e, h) channels in both leads. The sum over j is performed over the electron and hole channels only in the lead connected to the reservoir m for which the pumped charge is calculated. The function γ_{ij} is equal to $+1$ when the element $S_{i,j}$ of the scattering matrix corresponds to a process in which a current is pumped *from* lead m, $\gamma_{ij} = -1$ when a current is pumped *into* lead m. This difference in sign is due to the fact that electrons and holes contribute oppositely to the pumped charge. Since the electron and hole contributions to the pumped charge could exactly compensate each other, it is not obvious *a priori* whether a net charge can be pumped.

The problem of computing the pumped charge is then reduced to the calculation of the total scattering matrix S_{tot} of electrons and holes in the normal conductor bridge (see Fig. 1b), as a function of the parameters X_1 and X_2. The calculation is lengthy but conceptually straightforward (calculations were done using *Mathematica*™). We consider a perfectly symmetric configuration with two identical SNS junctions, which are also identically coupled to the normal metal bridge. For each junction, the coupling is described by a "beam-splitter" [11], whose scattering matrix ($S_{1,2}$) is assumed to be energy independent (*i.e.*, it is the same for electrons and holes). We have chosen the simplest expression compatible with unitarity and time reversal symmetry. The expression reads:

$$S_{1,2} = \begin{pmatrix} a & \sqrt{\frac{\varepsilon}{2}} & b & \sqrt{\frac{\varepsilon}{2}} \\ \sqrt{\frac{\varepsilon}{2}} & -a & \sqrt{\frac{\varepsilon}{2}} & -b \\ b & \sqrt{\frac{\varepsilon}{2}} & a & \sqrt{\frac{\varepsilon}{2}} \\ \sqrt{\frac{\varepsilon}{2}} & -b & \sqrt{\frac{\varepsilon}{2}} & -a \end{pmatrix}, \qquad (3)$$

where ϵ varies between 0 and 1/2 ($\epsilon/2$ is the probability for an incoming particle to be deflected towards one of the superconductors). The amplitudes for backscattering a and direct transmission b across the beam splitter satisfy the relations $a^2 + b^2 + \varepsilon = 1$ and $\varepsilon/2ab = 1$, imposed by unitarity. For every fixed value of ε two solutions, with $a > b$ and $b > a$, are possible and we considered both cases (for $\varepsilon = 1/2$ the two solutions coincide and $a = b = 1/2$).

Mixing of electrons and holes only occurs at the interface with the superconductors. Having assumed perfect transparency at the NS interfaces, the matrix describing Andreev reflection in the "vertical" branches of the circuit (see Fig.1) depends only on the phase χ of the superconducting order parameter. It reads:

$$S_{\rm AR} = \begin{pmatrix} 0 & r_{\rm he} \\ r_{\rm eh} & 0 \end{pmatrix} = \begin{pmatrix} 0 & -ie^{i\chi} \\ -ie^{-i\chi} & 0 \end{pmatrix}. \qquad (4)$$

To calculate the total scattering matrix of the device we first calculate the scattering matrix associated to transport across only one SNS junction. We then consider the two SNS junction connected in series, *i.e.* we consider all the multiple reflection processes in the normal metal bridge, taking into account the corresponding dynamical phases acquired by electrons and holes. The result is the scattering matrix $S_{tot}(X_1, X_2)$ that mixes the electron and hole channels in reflection and transmission.

Having determined S_{tot} we obtain the pumped charge from Eqs. 1 and 2. As shown in Fig. 2 we find that, unless $k_F L = n\pi$ (with n integer), the pumped charge is a non-zero, anti-symmetric, and 2π-periodic function of φ, as expected. The 2π periodicity in conjunction with the antisymmetry imply that the pumped charge has to vanish when $\varphi = \pm\pi$. This is the case since for $\varphi = \pi$ the trajectory in the space of the pumping parameters (X_1, X_2) does not enclose a finite area. In addition, the antisymmetry of Q_P with respect to φ also implies that the sign of the pumped current changes when reversing the sign of the relative phase of the two superconducting junctions. This results in the antisymmetry of the pumped current versus applied magnetic flux Φ (that determines the phase φ), and provides a distinctive feature that should facilitate the experimental identification of the phenomenon.

Fig. 3A, B, and C summarize the outcome of our calculations for the different cases $a > b$, $a = b$, and $a < b$. We first discuss the features of the results that are common to all three cases. We always find that the pumped charge does not depend on the separation W between the beam splitters and the superconducting leads (see Fig. 3). This is due to the phase conjugation [12] of electrons and holes at the Fermi energy, since the dynamical phase acquired by an electron propagating from the beam splitter to the superconducting interface is exactly compensated by the phase acquired by the Andreev reflected hole. In all cases the dependence of Q_P on L is periodic for all values of φ and ε, with period given by $k_F L = \pi$ (k_F is the Fermi wave vector). This implies that the pumped charge is sensitive to the geometry of the device on the scale of the Fermi wavelength λ_F, indicating that charge pumping in the device considered here is a sample specific phenomenon. That this should be so is not obvious a priori : owing to phase conjugation, one may have expected the pumped charge to show a component independent of the precise geometry of the device [13].

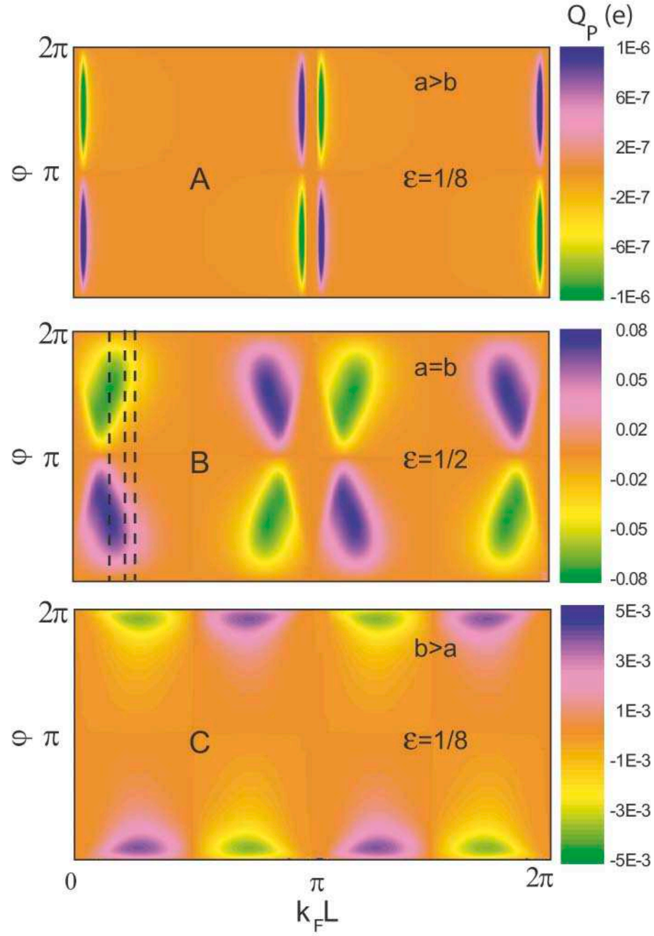

FIG. 3: Color scale plots of Q_P as a function of $k_F L$ and φ for three distinct cases of amplitude scattering on the beam splitter: $\varepsilon = 1/8$ for $a > b$ (top), $a = b = \varepsilon = 1/2$ (center) and $\varepsilon = 1/8$ for $b > a$ (bottom). The dashed lines in the center panel correspond to the curves shown in Fig. 2.

The magnitude of the calculated pumped charge strongly depends on ε. The maximum pumped charge is approximately 0.1 electron per cycle, for $\varepsilon = 1/2$ ($a = b$). For small ε, the magnitude of Q_P decreases with decreasing ε (and eventually vanishes for $\varepsilon = 0$) both when $a > b$ and $b > a$. The dependence of Q_P on φ and $k_F L$, however, is different in the two cases.

When $a > b$ and for small values of ε (*i.e.* $a \sim 1$) the bridge connecting the two SNS junctions is only weakly coupled to the reservoirs, because backscattering at the beam-splitters is the dominant process (see Fig. 3A). In this regime, sharply defined resonances appear in the conductance of the system when $k_F L = n\pi$ (with n integer), due to the presence of quasi-bound states in the bridge connecting the two SNS junctions. When $k_F L = n\pi$ the energy of a quasi-bound state aligns with the Fermi levels in the reservoirs. Interestingly, the pumped charge is also

significantly different from zero only when $k_F L$ is close to being a multiple of π. This suggests a close link between pumping and the presence of resonances due to quasi-bound states in the system, as already noted by others in different contexts[7]. This link is further supported by observing that increasing ε from 0 to 1/2 -corresponding to increasing the broadening of the quasi-bound states-results in a broader range of values of L for which charge pumping is observed (Fig. 3B).

In the case $b > a$, the behavior of the pumped charge for small values of ε is qualitatively different (see Fig. 3C). In this regime, the dominant process at the beam splitters is direct transmission. Therefore electrons and holes have only a small probability to be deflected from the normal bridge to the N/S interfaces. However, if they are deflected, they perform many Andreev reflections in one of the SNS junctions before they can escape again to the normal metal bridge. As a consequence, along the dominant trajectories responsible for pumping, electrons and holes have a large probability to acquire a phase $e^{iN\chi}$ (with different, and even large, integer values of N), rather than simply $e^{i\chi}$. This causes the phase dependence of the pumping signal to be richer in harmonics and, consequently, to exhibit very strong deviations from a simple sine dependence, as seen from Fig. 3C.

Having established the occurrence of adiabatic quantum pumping, we briefly discuss some of the advantages of the proposed device. The use of the *ac* Josephson effect to generate the time dependence of the pumping parameters (the superconducting phases in our case) should allow operation at frequencies of several hundreds GHz. In fact, with superconductors such as Nb, NbN, or NbTiN, values for the superconducting gap Δ corresponding to frequencies in excess of 1 THz are possible, so that our superconducting pump can operate at a few hundreds GHz when the voltages applied across the SNS junctions is still sufficiently lower than Δ. At a Josephson frequency of 100 GHz, the pumped current can exceed 1 nA, which is easily measurable. Note that, since the pumping parameters are coupled to the electron-hole wave functions via Andreev reflection, the coupling will remain good at these high frequencies. In addition, the fact that no external microwave signals need to be fed into the circuit to drive the pumping parameters implies that only a negligible high-frequency power will be irradiated, thereby minimizing the possibility of rectification effects known to cause problems in other systems[5]. For the practical realization of the proposed superconducting pump we suggest the use of a ballistic InAs-based two-dimensional electron gas as normal conductor. Present technology enables the reduction of the number of conducting channels to ≈ 10 [14], which is important since the predicted effect is of the order of one channel. The use of InAs also enables the realization of the needed highly transparent contacts to superconductors [15]. Furthermore, in ballistic devices in which the distance between the two SNS junction is $L \simeq 1\mu$m, the typical propagation time in the device will be of the order of L/v_F 10^{-12} s ($v_F \simeq 10^6$ m/s is typically realized in InAs heterostructures). This is ten times faster than the period of an *ac* pumping signal oscillating at 100 GHz, ensuring that the dwell time of electrons is much shorter than the period of the ac pumping signal, as it is needed for the device to operate in the adiabatic regime.

We gratefully acknowledge M. Blauboer, Y.V. Nazarov, Y. Tokura and P.W. Brouwer for useful discussions. This research is financially supported by the Dutch Foundation for Fundamental Research (FOM) and NWO (Vernieuwingsimpuls 2000).

[1] D. J. Thouless, Phys. Rev. B**27**, 6083 (1983).
[2] F. Zhou, B. Spivak and B. L. Altshuler, Phys. Rev. Lett. **82**, 608 (1999).
[3] P. W. Brouwer, Phys. Rev. B **58**, 10135(R) (1998).
[4] M. Switkes, C. M. Marcus, K. Campman, A. C. Gossard, Science **283** 1905 (1999).
[5] P. W. Brouwer, Phys. Rev. B **63** 121303(R) (2001); L. DiCarlo and C. Marcus and J. S. Harris, Jr., Phys. Rev. Lett. **91**, 246804 (2003).
[6] N. B. Kopnin, A. S. Mel'nikov and V. M. Vinokur Phys. Rev. Lett. **96**, 146802 (2006); M. Moskalets and M. Büttiker, Phys. Rev. B **64**, 201305(R) (2001); J. E. Avron, A. Elgart, G. M. Graf, and L. Sadun, Phys. Rev. B **62**, 10618(R) (2000); C. Benjamin, Europ. Phys. J. B **52**, 403 (2006); M. Governale *et al.*, Phys. Rev. Lett. **95**, 256801 (2005).
[7] Y. Wei, J. Wang and H. Guo, Phys. Rev. B *62*, 9947 (2000); O. Entin-Wohlman and A. Aharony, Phys. Rev. B **66**, 035329 (2002).
[8] J. Wang *et al.*, Appl. Phys. Lett. **79**, 3977 (2001); M. Blaauboer, Phys. Rev. B **65**, 235318 (2002); F. Taddei, M. Governale and R. Fazio, Phys. Rev. B **70**, 052510 (2004).
[9] A. F. Andreev, Sov. Phys. JETP **19**, 1228 (1964).
[10] K. K. Likharev, *Dynamics of Josephson Junctions and Circuits*, New York, Gordon and Breach (1986).
[11] M. Büttiker, IBM J. Res. Dev. **32**, 63 (1988).
[12] H. Takayanagi and B. J. van Wees in *Mesoscopic Electron Transport* (NATO Science Series E: Applied Sciences **345** 469-501), Eds. L. L. Sohn, G. Schön and L. P. Kouwenhoven, Dordrecht, Kluwer Academic (1997).
[13] An "ensemble averaged" component of pumped current has been found previously by Zhou (F. Zhou, Int. Jour. Mod. Phys. B **15**, 117 (2001)), who had to assume electron-hole asymmetry on the scale of the superconducting gap. Since this asymmetry is usually small, we expect that in real devices the sample specific effect that we calculate here is much larger in amplitude than the one discussed by Zhou.
[14] A. F. Morpurgo, B. J. van Wees, T. M. Klapwijk and G. Borghs Appl. Phys. Lett. **70**, 1435 (1997).
[15] J. P. Heida, B. J. van Wees, T. M. Klapwijk and G. Borghs Phys. Rev. B **57**, 5618(R) (1998).